

Tangent to a Circle — Theorem 10.1

NCERT Class 10 Maths · Chapter 10 Circles · Section 10.2 · Exercise 10.1

At the point of contact, $OP \perp XY$ — every tangent question in the chapter is that one 90° in disguise.

The four names, and the two students swap

Name	What it is	On the figure
Point of contact	the single point where the tangent meets the circle	P
Radius to the point of contact	centre $\rightarrow$ P; it is a leg of the right triangle	$OP = r$
Length of the tangent	P $\rightarrow$ the external point; the other leg	PQ
Distance to the external point	centre $\rightarrow$ Q; always the longest side	OQ (hypotenuse)

The two that get swapped: OP is the radius, PQ is the tangent length — different sides. And OQ is *never* a leg. Getting either wrong flips a subtraction into an addition.

Theorem 10.1, its proof in one sentence, and what it builds

Theorem 10.1. *The tangent at any point of a circle is perpendicular to the radius through the point of contact.* If XY touches a circle of centre O at P, then $OP \perp XY$, so $\angle OPX = \angle OPY = 90^\circ$.

Why. Every point of XY except P lies *outside* the circle, so OP is the **shortest** distance from O to the line — and the shortest distance from a point to a line is the perpendicular.

What it builds: a right triangle OPQ, right-angled at P, so $OQ^2 = OP^2 + PQ^2$ and therefore $PQ = \sqrt{OQ^2 - r^2}$.

Two free consequences: the perpendicular to a tangent at its point of contact **passes through the centre**; and the tangents at the two ends of a **diameter are parallel**.

Spotting it in the question:

- “a tangent at P” or “touches the circle at P” $\rightarrow$ mark **90° at P** before reading further.
- “meets a line through the centre at Q” $\rightarrow$ OQ is the **hypotenuse**, so you will **subtract**.
- “the length of the tangent” $\rightarrow$ that is **PQ**, contact point to external point — never the radius.

Where this sits in NCERT and CBSE

Question	Answer
Which section?	NCERT Chapter 10 <i>Circles</i> , Section 10.2 — Tangent to a Circle
Which exercise?	Exercise 10.1, 4 questions. Mind the offset: Section 10.2, Exercise 10.1
How many marks?	Unit IV Geometry, 15 of the 80 theory marks, shared with Triangles
Is the proof asked?	Yes — CBSE lists this result as one to prove

The CBSE syllabus, word for word: “1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact. 2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.” Those two are the **whole** of Class 10 Circles — angles in the same segment and cyclic quadrilaterals are Class 9 and are not examined here.

Key takeaways

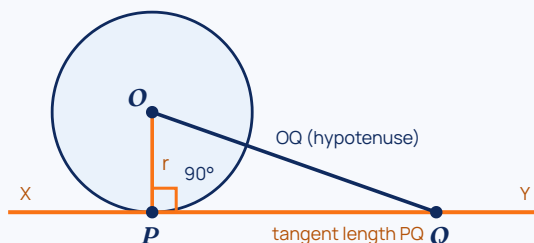
- A **tangent** touches a circle at **exactly one point**, the point of contact, and never crosses inside.
- **Theorem 10.1:** tangent $\perp$ radius at the point of contact — the angle is **90°** .
- **Proof in one line:** every other point of the tangent is outside the circle, so OP is the shortest distance, so it is perpendicular.
- **Tangent length:** $PQ = \sqrt{OQ^2 - r^2}$ — OP and PQ are the legs, **OQ is the hypotenuse**.
- **Exactly one** tangent at any point *on* a circle; two tangents need an **external** point.
- **NCERT** Ch. 10, **Section 10.2, Exercise 10.1**; **CBSE** Unit IV Geometry, **15 of 80**, marked **(Prove)**.

Write it the way it is marked

The 3-mark proof, the Exercise 10.1 numerical, and where the marks actually go

3 MARKS · STATEMENT AND PROOF

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.



The right angle sits at **P**, the point of contact — not anywhere else on the tangent.

Given: A circle with centre O and a tangent XY touching it at P.

To prove: $OP \perp XY$.

Construction: Take any point Q on XY other than P, and join OQ.

Proof:

- Q lies on the tangent, so Q lies **outside** the circle (if Q were inside, XY would cut the circle twice and be a secant)
- $\therefore OQ > OP$ (a point outside the circle is farther from O than the radius)
- This holds for **every** point of XY except P, so **OP is the shortest** distance from O to XY
- The shortest distance from a point to a line is the **perpendicular** distance
- $\therefore OP \perp XY$. Hence proved.

NCERT EXERCISE 10.1, Q5

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12$ cm. Find PQ.

Step 1 — name the right angle. PQ is a tangent and OP is the radius to the point of contact, so by Theorem 10.1 $OP \perp PQ$. Triangle OPQ is right-angled at P. (This line is a mark.)

Step 2 — label the sides. $OP = r = 5$ cm (leg), PQ = tangent length (leg), **$OQ = 12$ cm is the hypotenuse.**

Step 3 — Pythagoras. $OQ^2 = OP^2 + PQ^2 \Rightarrow PQ^2 = 12^2 - 5^2 = 144 - 25 = 119$.

Step 4 — answer. $PQ = \sqrt{119} \text{ cm} \approx 10.9 \text{ cm}$.

Check: the textbook offers **13 cm** as an option — that is $\sqrt{12^2 + 5^2}$, what you get by *adding*. If your tangent length comes out longer than OQ, you have used OQ as a leg.

WHERE THE MARKS ACTUALLY GO

- The **90°** is placed at the **wrong vertex** — it belongs at P, where the radius meets the tangent, nowhere else.
- The **reason is missing** — “ $PQ = \sqrt{(OQ^2 - r^2)}$ ” with no “ $OP \perp PQ$ by Theorem 10.1” above it drops the method mark.
- OQ treated as a leg** — adding instead of subtracting; the answer is then bigger than the hypotenuse.
- The **proof is asserted, not argued** — without the line “every other point of XY is outside the circle” there is nothing to prove from.

NOW TRY THIS ONE

A tangent from an external point Q touches a circle of centre O at P. The radius is **12 cm** and the tangent length **PQ = 35 cm**. Find **OQ** — and write the reason line before you use Pythagoras.

Answer: $OP \perp PQ$ (Theorem 10.1), so $OQ^2 = 12^2 + 35^2 = 144 + 1225 = 1369$, giving **OQ = 37 cm**. Here you *add*, because the unknown is the hypotenuse — which is exactly why naming the sides before calculating is worth doing every time.

Make the right angle automatic — on examfront

- Chapter 1 is free** — start today, no payment, and see how the practice works.
- Mistake identification** — we name the slip: wrong vertex, wrong side, missing reason line.
- Anytime tests** — drill tangent questions whenever you have twenty minutes.
- Weekly tests** — keep Circles warm while you move on to the next chapter.
- Mock tests before the exam** — full papers, real timing, so board day is not the first time.

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