

Number of Zeroes of a Polynomial from a Graph

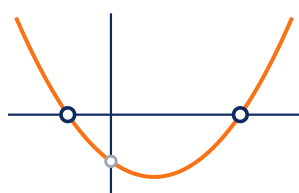
2-page summary – find the x-axis, count every meeting point, check the count against the degree.

Number of zeroes of $p(x)$ = number of distinct points where the graph of $y = p(x)$ meets the x-axis.
 A crossing counts 1 · a touch counts 1 · a graph that never meets the x-axis has 0 zeroes.

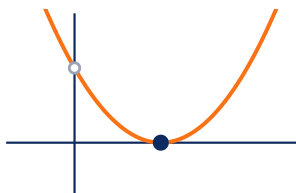
The words you need

Zero of $p(x)$	A number k with $p(k) = 0$. On the graph it is the point $(k, 0)$.
x-axis	The line $y = 0$ – the only line where zeroes are read.
Crossing point	The curve passes through the x-axis. One zero.
Touching point	The curve meets the x-axis and turns back. Still one zero, not two.
y-intercept	The point $(0, p(0))$. Not a zero, unless $p(0) = 0$ and the graph passes through the origin.

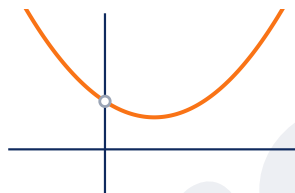
Four pictures, four answers



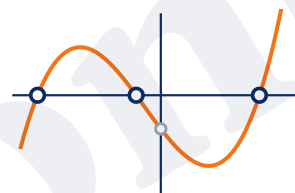
Crosses twice → **2**



Touches once → **1**



Never meets → **0**



Three crossings, one near the edge → **3**

Drawn from $x^2 - 2x - 3$, $(x - 2)^2$, $x^2 - 2x + 3$ and $(x + 2.5)(x + 0.5)(x - 2)$. ○ crossing ● touching ○ y-intercept, never counted in these four.

The 3-step method, and the trap inside each step

Step	The trap
1. Find the x-axis ($y = 0$)	Reading the y-axis crossing as a zero
2. Mark every meeting, edge to edge	Stopping at the points already circled or labelled
3. Count the points	Counting bends, or counting a touch as two

Degree check: linear at most 1, quadratic at most 2, cubic at most 3, degree n at most n zeroes – *at most*, never *exactly*. Three crossings on a graph labelled quadratic means recount.

Where it sits in NCERT and CBSE

NCERT: Chapter 2, Section 2.2, *Geometrical Meaning of the Zeroes of a Polynomial*. Example 1 reads six graphs (Fig. 2.9); **Exercise 2.1** is one question on six more (Fig. 2.10) – answers **0, 1, 3, 2, 4, 3**. Section 2.2 but Exercise 2.1: Section 2.1 is the introduction.

CBSE: "Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials." Unit II Algebra = **20 of 80** marks. Division algorithm removed. Usually a **one-mark Section A** question – no method marks.

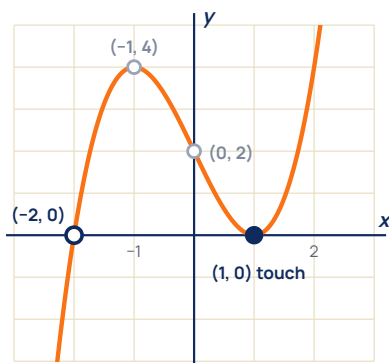
Key takeaways

- **Count x-axis meeting points** – that count is the number of zeroes.
- **Cross = 1, touch = 1, no meeting = 0**. A touch is never two.
- **Scan edge to edge**. An unlabelled crossing near the edge still counts.
- **The y-axis crossing is $p(0)$** – a zero only when the graph goes through the origin.
- **Degree n allows at most n zeroes**; fewer is normal, more means a miscount.
- **NCERT Exercise 2.1** is this skill exactly; it is worth one mark in Section A.

Worked Example: Read One Graph the Way the Examiner Does

WORKED EXAMPLE · 1 MARK, SECTION A STYLE

The graph of $y = p(x)$, where $p(x)$ is a cubic polynomial, is shown. How many zeroes does $p(x)$ have?



Step 1 – Find the x-axis. The horizontal line $y = 0$.

Step 2 – Mark every meeting, edge to edge. At $(-2, 0)$ the curve **crosses**. At $(1, 0)$ it comes down, **touches** and turns back up.

Step 3 – Rule out the distractors. $(0, 2)$ is where the curve meets the **y-axis**: that is $p(0) = 2$, not a zero. $(-1, 4)$ is the top of a **bend**: not a zero.

Step 4 – Count. Two points on the x-axis, so two zeroes.

Check: $p(x) = x^3 - 3x + 2 = (x - 1)^2(x + 2)$. $p(-2) = -8 + 6 + 2 = 0$ and $p(1) = 1 - 3 + 2 = 0$, but $p(0) = 2$. A cubic allows at most 3, so 2 passes.

Answer: 2 zeroes – fewer than the degree, which is allowed.

DOES THIS POINT COUNT?

Point	What it is on the graph	Counts as a zero?
$(-2, 0)$	The curve crosses the x-axis	Yes – one zero
$(1, 0)$	The curve touches the x-axis and turns back	Yes – one zero, not two
$(0, 2)$	Where the curve meets the y-axis	No – it is $p(0) = 2$
$(-1, 4)$	The top of a bend (a turning point)	No – bends are not zeroes
$(0, 0)$	Any graph that passes through the origin	Yes – on the x-axis too; count once

CHECK YOUR NCERT EXERCISE 2.1 (FIG. 2.10)

(i)	(ii)	(iii)	(iv)	(v)	(vi)
0	1	3	2	4	3

Work all six from the textbook first. If one of yours disagrees, re-scan that graph from its left edge to its right edge before you change the answer.

WHERE THE ONE MARK IS LOST

Counted the y-intercept	$(0, p(0))$ is on the y-axis; only x-axis points are zeroes.
Counted a touch as two	You are counting points, and a touch is one point.
Missed an edge crossing	The labelled points are not the whole answer. Scan the curve end to end.
Counted above the degree	Three zeroes for a quadratic is always a recount.

NOW TRY THIS ONE

A cubic's graph rises from below, **crosses the x-axis at the origin**, climbs to a peak at $(1, 4)$, comes back down to **just touch** the x-axis at $(3, 0)$, then rises again. (1) How many zeroes does the polynomial have? (2) The graph meets the y-axis at the origin – does that point count?

Make graph questions automatic with examfront

- **Free Chapter 1** – try the complete practice flow at no cost before you commit.
- **Mistake identification** – see exactly which point you miscounted, and why.
- **Anytime tests** – a quick graph set whenever you have ten spare minutes.
- **Weekly tests** – keep Polynomials sharp while you move through Algebra.
- **Mock tests before the exam** – full CBSE-pattern papers so Section A feels routine.

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