

Number of Tangents from a Point on a Circle

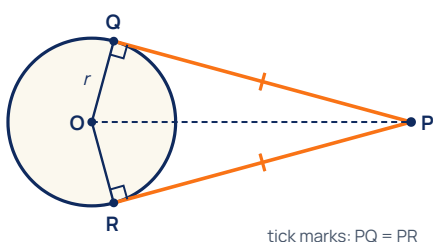
The 0-1-2 rule, the length of a tangent and Theorem 10.2 – the summary on two pages.

0 · 1 · 2 – where the point sits decides how many tangents you can draw. From an outside point the two tangents are **equal**: $PQ = PR$, and each one satisfies $PQ^2 = OP^2 - r^2$.

Three positions, three answers

Where the point is	Tangents	Why
Inside the circle	0	Every line through it cuts the circle twice – a secant
On the circle	1	The tangent must be perpendicular to OP; only one such line
Outside the circle	2	Two lines just touch it, at Q and R, and $PQ = PR$

The right triangle behind every tangent length



tick marks: $PQ = PR$

Length of a tangent from P: the distance from P to the point of contact Q.

The radius OQ meets the tangent at 90° (Theorem 10.1), so triangle OQP is right-angled at Q, and **OP is always the hypotenuse**:

$$PQ = \sqrt{OP^2 - r^2}$$

Two more facts from the same figure: **OP bisects $\angle QPR$** , and $\angle QPR + \angle QOR = 180^\circ$, because the quadrilateral OQPR already holds two right angles.

Quick use. Ex 10.2 Q1: $OP = 25$ cm and $PQ = 24$ cm, so $r = \sqrt{(625 - 576)} = 7$ cm. Ex 10.2 Q2: the angle at the centre is 110° , so the angle between the tangents is $180^\circ - 110^\circ = 70^\circ$.

Theorem 10.2 – the proof, in the CBSE shape

Line	What you write	Reason
Given / To prove	Tangents PQ, PR from external point P; prove $PQ = PR$	—
Construction	Join OQ, OR and OP	creates the two triangles
Right angles	$\angle OQP = \angle ORP = 90^\circ$	Theorem 10.1
Three conditions	$OQ = OR$; OP common; both right-angled	radii; common side
Conclusion	$\triangle OQP \cong \triangle ORP$, so $PQ = PR$	RHS, then CPCT

Pythagoras gives the same result in one line: $PQ^2 = OP^2 - OQ^2 = OP^2 - OR^2 = PR^2$, so $PQ = PR$.

NCERT and CBSE. This is NCERT Chapter 10, **Section 10.3** – but its exercise is **Exercise 10.2 (13 questions)**; Exercise 10.1 (4 questions) belongs to Section 10.2. CBSE lists Circles in **Unit IV Geometry, 15 of 80 marks** (with Triangles) and writes this result as “(Prove) The lengths of tangents drawn from an external point to a circle are equal.” Constructions has been removed.

Key takeaways

- ▶ **0, 1, 2** tangents from a point inside, on, or outside the circle.
- ▶ **Tangent length** $= \sqrt{OP^2 - r^2}$; OP is the hypotenuse, never a leg.
- ▶ **Theorem 10.2**: tangents from an external point are equal, $PQ = PR$.
- ▶ **Proof**: Construction line, two right angles, RHS, then CPCT.
- ▶ **Angle fact**: angle between the tangents + angle at the centre = 180° .
- ▶ **Syllabus**: Section 10.3, Exercise 10.2; CBSE says “(Prove)”, so write the proof.

Equal Tangents, Put to Work

Label the tangent segments first. Almost every question in this exercise turns on that one move.

NCERT EX 10.2 Q12 · WORKED IN FULL

Question. A triangle ABC is drawn to circumscribe a circle of radius 4 cm. The circle touches BC at D with $BD = 8$ cm and $DC = 6$ cm. Find AB and AC.

Step 1 – label the equal tangents. Let the circle touch CA at E and AB at F. From each vertex the two tangents are equal (Theorem 10.2): $BF = BD = 8$, $CE = CD = 6$, and $AE = AF = x$.

Step 2 – write the sides. $BC = 14$, $CA = x + 6$, $AB = x + 8$. Semi-perimeter $s = x + 14$.

Step 3 – area two ways. Joining O to A, B and C splits the triangle into three with height 4: $\text{area} = \frac{1}{2} \times 4 \times (14 + (x + 6) + (x + 8)) = 4(x + 14)$. By Heron's formula, $\text{area} = \sqrt{(x + 14)(x)(8)(6)} = \sqrt{48x(x + 14)}$.

Step 4 – equate and solve. $48x(x + 14) = 16(x + 14)^2 \Rightarrow 3x = x + 14 \Rightarrow x = 7$.

AB = 7 + 8 = 15 cm and AC = 7 + 6 = 13 cm.

Check: sides 13, 14, 15 give $s = 21$ and $\text{area} \sqrt{(21 \times 8 \times 7 \times 6)} = 84 \text{ cm}^2$; $\text{radius} = \text{area} / s = 84/21 = 4$ cm, exactly the given radius.

NCERT EX 10.2 Q8 · THE PROOF SHAPE

Prove that for a quadrilateral ABCD circumscribing a circle, $AB + CD = AD + BC$.

Let the circle touch AB, BC, CD, DA at P, Q, R, S. By Theorem 10.2: **AP = AS, BP = BQ, CR = CQ, DR = DS.**

Add them: $AP + BP + CR + DR = AS + BQ + CQ + DS$.

Regroup: $(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$, that is **AB + CD = AD + BC**. Hence proved.

WHERE THE MARKS ACTUALLY GO

No tangent segments labelled

the equal pairs are the method; without them the working has nothing to mark

Construction line missing

the right angles at Q and R only exist once OQ and OR are joined

RHS with one condition dropped

write equal radii, common OP and both right angles, each with its reason

Radius used as the hypotenuse

OP, centre to external point, is always the longest side

EXERCISE 10.2 AT A GLANCE · ALL 13 QUESTIONS

Shape of question	Questions	The move that unlocks it
Find a length	Q1, Q6, Q7	right angle at the point of contact
Angle between two tangents	Q2, Q3, Q10	the two angles add to 180°
Tangent and radius proofs	Q4, Q5	Theorem 10.1: radius meets tangent at 90°
Figures around a circle	Q8, Q9, Q11, Q12, Q13	label equal tangents from each vertex

NOW TRY THESE TWO

(1) Ex 10.2 Q3: tangents PA and PB from P are inclined to each other at 80° . Find $\angle POA$.

(2) Ex 10.2 Q6: the tangent from a point A, 5 cm from the centre, is 4 cm long. Find the radius.

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