

Nature of Roots of a Quadratic Equation

Class 10 · Quadratic Equations · CBSE & NCERT — 2-page summary

$D = b^2 - 4ac$. One number, built from a , b and c , tells you what *kind* of roots $ax^2 + bx + c = 0$ has — before you solve it. Read its sign, not its size.

The three cases

Discriminant	Nature of the roots	What the parabola does	Example
D is positive	Two distinct real roots	Cuts the x-axis at two points	$2x^2 - 6x + 3 = 0$, $D = 12$
D is zero	Two equal real roots, both $-b/2a$	Touches the x-axis at one point	$9x^2 - 6x + 1 = 0$, $D = 0$
D is negative	No real roots	Never meets the x-axis	$2x^2 - 4x + 3 = 0$, $D = -8$

Write the exam words exactly: **“two equal real roots”** when D is zero (not “one root”), and **“no real roots”** when D is negative (not “no roots”).

Three steps, and the trap inside each one

Step	What you write	The trap
1. Standard form	$ax^2 + bx + c = 0$; read off a , b , c with their signs	a must not be zero — if the x^2 term vanishes there is no discriminant
2. Discriminant	$D = b^2 - 4ac$, then substitute	b^2 is never negative: $b = -4$ gives 16, not -16
3. Read the sign	Name the nature; find the roots only if asked	$-4ac$ flips sign with c : $c = -60$ gives +240

The quadratic formula is $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$, so the discriminant *is* the part under the root.

The three ways a “find the value of k ” question is worded

The question says	You write	What comes out
“has two equal roots”	$D = 0$	An equation in k — then reject any k making $a = 0$
“has two distinct real roots”	D is positive	A range of k , so quote the condition, not one value
“is such a design possible?”	the sign of D	A yes or no — the sign alone answers it

Where this sits in NCERT and CBSE

The nature of roots is the **last teaching section of NCERT Chapter 4, Quadratic Equations**, drilled in **Exercise 4.3 (5 questions)**. The chapter has three exercises now: 4.1 (2 questions, is it quadratic and represent this situation), 4.2 (6 questions, factorisation) and 4.3. The section is the fourth of the chapter but its exercise is numbered 4.3, because Section 4.1 is the introduction and carries no exercise.

CBSE, word for word: “Standard form of a quadratic equation $ax^2 + bx + c = 0$, ($a \neq 0$). Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.” Two consequences: **($a \neq 0$) is in the syllabus itself**, so rejecting a k that kills the x^2 term is a real mark; and **“only real roots” means “no real roots” is a complete answer** — nothing about imaginary roots is expected. **Completing the square was removed in the rationalisation**, so factorisation and the quadratic formula are the only algebraic methods examined. Quadratic Equations sits in **Algebra, 20 of the 80** theory marks.

Key takeaways

- **One number decides it.** $D = b^2 - 4ac$, computed from the standard form, and you never solve the equation.
- **Positive, zero, negative** map to two distinct real roots, two equal real roots (both $-b/2a$), and no real roots.
- **Equal roots means $D = 0$.** That single equation is what every “find the value of k ” question is asking for.
- **Reject any k that makes $a = 0$.** The syllabus writes the condition as ($a \neq 0$), so the rejection line carries a mark.
- **Show the discriminant line.** Naming the nature without $D = b^2 - 4ac$ and the substitution is an incomplete answer.
- **NCERT Exercise 4.3, 5 questions**, inside Algebra (20 of 80). Real roots only — and completing the square is off the syllabus.

NCERT EX 4.3, Q2(i) – 3 MARKS

Find the values of k so that $2x^2 + kx + 3 = 0$ has two equal roots.

Step 1 ► Standard form. It is already standard: $a = 2$, $b = k$, $c = 3$, and a is not zero whatever k does.

Step 2 ► Write the condition. Two equal roots means $D = 0$, so $b^2 - 4ac = 0$.

Step 3 ► Substitute. $k^2 - 4(2)(3) = 0$, so $k^2 - 24 = 0$ and $k^2 = 24$.

Step 4 ► Solve. $k = \pm\sqrt{24} = \pm 2\sqrt{6}$, that is $k \approx 4.90$ or $k \approx -4.90$.

Check: with $k = 2\sqrt{6}$ the repeated root is $-b/2a = -2\sqrt{6}/4 = -\sqrt{6}/2 \approx -1.22$, and substituting gives $2(1.5) - 6 + 3 = 0$. **Both** values are kept here – unlike Q2(ii), where $a = k$ and $k = 0$ has to be thrown out. Deciding whether a value survives is a mark on its own.

NCERT EX 4.3, Q5 – THE DESIGN THAT WORKS

Is it possible to design a rectangular park of perimeter 80 m and area 400 m²?

Step 1 ► Name and translate. Length l , breadth b . Perimeter 80 gives $l + b = 40$, so $b = 40 - l$. Area 400 gives $l(40 - l) = 400$.

Step 2 ► Standard form. $40l - l^2 = 400$, so $l^2 - 40l + 400 = 0$; $a = 1$, $b = -40$, $c = 400$.

Step 3 ► Discriminant. $D = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$.

Step 4 ► Answer the question asked. $D = 0$, so real roots exist and the design is **possible**. The repeated root is $l = -b/2a = 40/2 = 20$ m, so breadth = 20 m too.

Check: perimeter $2(20 + 20) = 80$ m ✓ and area $20 \times 20 = 400$ m² ✓. $D = 0$ is telling you something real here: there is exactly **one** such rectangle, and it happens to be a square.

NCERT EX 4.3, Q4 – THE ONE THAT IS IMPOSSIBLE

The ages of two friends sum to 20. Four years ago the product of their ages was 48. Is this possible?

Translate. Ages x and $20 - x$. Four years ago: $(x - 4)$ and $(16 - x)$, so $(x - 4)(16 - x) = 48$.

Standard form. $20x - x^2 - 64 = 48$, which rearranges to $x^2 - 20x + 112 = 0$.

Discriminant. $D = (-20)^2 - 4(1)(112) = 400 - 448 = -48$, which is negative.

Answer. No real roots, so the situation is **not possible**. Stop there – that is the full answer.

This is why the discriminant earns its keep: you have answered a real question **without solving anything**, and a negative D is a result, not a dead end. Students lose marks here by assuming they have made an error and hunting for roots that do not exist.

WHERE THE MARKS ACTUALLY GO

What students do	What it costs
Name the nature with no D line	The substitution into $b^2 - 4ac$ is itself a mark
Write “one root” or “no roots”	The exam wants “two equal real roots” and “no real roots”
Keep a k that makes $a = 0$	The equation stops being quadratic; the rejection line is a mark
Solve when only the nature is asked	Costs minutes, earns nothing, and invites a fresh arithmetic slip

NOW TRY THIS ONE

Find the nature of the roots of $3x^2 - 2x + 1/3 = 0$, and find them if they are real.

Answer at the foot of the page. Watch the fraction – it belongs in c , not under the root.

Make the discriminant automatic on examfront

- **Chapter 1 is free** – start today, no card, and see how the practice works before you commit.
- **Mistake Identification** names the exact slip – the sign of b^2 , the sign of $-4ac$, or the k you should have rejected.
- **Anytime tests** on nature of roots alone, so you drill the one line that carries the mark.
- **Weekly tests** mix Quadratic Equations with the rest of Algebra, the way the paper actually does.
- **Full mock tests before the exam** – 38 questions, 80 marks, three hours, timed like the real thing.

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