

Irrational Numbers — Class 10 Summary

NCERT Chapter 1, Section 1.3 "Revisiting Irrational Numbers" · Exercise 1.2 · Unit I, Number Systems (6 of 80 marks)

Irrational: **cannot** be written as a/b with a, b integers and $b \neq 0$ — the decimal **never ends and never repeats**

Rational or irrational? The test that decides

Number	Which is it?	Why
$\sqrt{4}, \sqrt{9}, \sqrt{25}$	Rational	Perfect squares — they come out whole (2, 3, 5)
$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{8}$	Irrational	Not perfect squares — the root never resolves
$0.75 \cdot 0.333\dots$	Rational	Terminating, or repeating in a block ($3/4, 1/3$)
$22/7$	Rational	It is a fraction. It approximates π ; it is not π
$\pi = 3.14159\dots$	Irrational	Non-terminating and non-repeating
$5 - \sqrt{3} \cdot 3\sqrt{2}$	Irrational	Rational $\pm$ irrational; non-zero rational $\times$ irrational

The shortcut that wins MCQs: the square root of a perfect square is rational; the square root of anything else is irrational. And beware the reverse trap — **irrational + irrational is not always irrational**: $\sqrt{2} + (-\sqrt{2}) = 0$, which is rational.

The master key, and the five beats of the proof

Master key: if a prime p divides a^2 , then p divides a . It comes from the Fundamental Theorem of Arithmetic — squaring a number creates no new prime factors, it only doubles the ones already there. Sense-check: $6^2 = 36 = 2^2 \times 3^2$ — still just 2 and 3.

Beat	What you write	Why it earns the mark
Assume	$\sqrt{2} = a/b$, a and b coprime	Without <i>coprime</i> there is no contradiction later
Square	$a^2 = 2b^2$	Turns a surd into an integer statement
Even (a)	2 divides $a^2 \Rightarrow a$ is even; $a = 2c$	The master key in use — name it, never skip it
Even (b)	$4c^2 = 2b^2 \Rightarrow b^2 = 2c^2 \Rightarrow b$ is even	The halving line is a real step; write it out
Contradiction	2 divides a and b — but they are coprime	Name it, then conclude — the closing mark

What NCERT and CBSE actually require

NCERT sets the scope. Chapter 1 now runs 1.1 Introduction, **1.2 Fundamental Theorem of Arithmetic**, **1.3 Revisiting Irrational Numbers**. Euclid's division lemma has been removed — check the contents page of a borrowed book first.

Numbering trap: the section is 1.3, the exercise is 1.2. Exercise 1.2 asks you to prove $\sqrt{5}$ is irrational, then $3 + 2\sqrt{5}$, then $1/\sqrt{2}$, $7\sqrt{5}$ and $6 + \sqrt{2}$.

CBSE narrows it further — use this as a self-check. The syllabus names the requirement almost word for word: the Fundamental Theorem of Arithmetic, and **proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$** . Three surds, named. Produce those three cleanly, handle one rational-plus-surd combination, and you have covered what the board has said it will ask.

CBSE sets priority by unit, never by chapter. Real Numbers sits in Unit I, Number Systems, worth **6 of the 80** theory marks (Number Systems 6, Algebra 20, Coordinate Geometry 6, Geometry 15, Trigonometry 12, Mensuration 10, Statistics & Probability 11 = 80, plus 20 internal). Any chapter-level percentage you find online is somebody's estimate, not an official figure.

How it arrives. The paper is 38 questions, 80 marks, three hours, five sections with no overall choice: A 20×1, B 5×2, C 6×3, D 4×5, E 3×4. An irrationality proof is normally a **2-mark Section B or 3-mark Section C** item, occasionally a 1-mark MCQ asking you to *spot* an irrational number.

Key takeaways

- Definition first.** Irrational means no a/b exists — decimal never ends, never repeats.
- One master key opens every proof:** prime p divides $a^2 \Rightarrow p$ divides a .
- The recipe never changes** — only the prime does.
- Say "coprime" in line one.** Skip it and the contradiction at the end proves nothing.
- For combinations, isolate the surd** — then the contradiction appears on its own.
- Six marks — but the most reliable six in the paper.** Few concepts, fixed proofs, no diagram.

The Proofs, Written the Way CBSE Marks Them

One full 3-mark proof, the same recipe on a different prime, an NCERT combination question — and where the marks sit

3 MARKS · SECTION C

Prove that $\sqrt{2}$ is irrational.

Assume, to the contrary, that $\sqrt{2}$ is rational.

Then $\sqrt{2} = a/b$ where a, b are integers, $b \neq 0$, and a, b are **coprime**. — *the assumption line*

Squaring: $2 = a^2/b^2 \Rightarrow a^2 = 2b^2$... (1)

$\Rightarrow 2$ divides $a^2 \Rightarrow 2$ divides a . — *by the master key* Let $a = 2c$.

Substituting in (1): $(2c)^2 = 2b^2 \Rightarrow 4c^2 = 2b^2 \Rightarrow b^2 = 2c^2$

$\Rightarrow 2$ divides $b^2 \Rightarrow 2$ divides b .

So 2 divides both a and b — but a and b are coprime. **Contradiction.**

$\therefore$ our assumption is wrong. **Hence $\sqrt{2}$ is irrational.**

SAME RECIPE, PRIME 3

Prove that $\sqrt{3}$ is irrational. Nothing changes but the prime.

Assume $\sqrt{3} = a/b$, a and b coprime $\Rightarrow a^2 = 3b^2 \Rightarrow 3$ divides $a^2 \Rightarrow 3$ divides a . Let $a = 3c$.

Then $9c^2 = 3b^2 \Rightarrow b^2 = 3c^2 \Rightarrow 3$ divides $b^2 \Rightarrow 3$ divides b .

Both share the factor 3, contradicting coprime. **Hence $\sqrt{3}$ is irrational.** *Swap 3 for 5 and you have $\sqrt{5}$ — the third proof CBSE names.*

NCERT EXERCISE 1.2 Q2 · 2 MARKS

Prove that $3 + 2\sqrt{5}$ is irrational. This is the combination type — and it leans on the proof above rather than repeating it.

Assume $3 + 2\sqrt{5}$ is rational. Then $3 + 2\sqrt{5} = a/b$ with a, b integers, $b \neq 0$.

Rearranging: $2\sqrt{5} = a/b - 3 \Rightarrow \sqrt{5} = (a - 3b) / 2b$

a, b are integers, so $(a - 3b)/2b$ is a **rational** number.

That would make $\sqrt{5}$ rational — but $\sqrt{5}$ is irrational (proved above). **Contradiction.**

$\therefore 3 + 2\sqrt{5}$ is irrational. *Engine: isolate the surd on one side; everything else is visibly rational.*

WHERE THE 3 MARKS ACTUALLY GO

What students write	What it costs
"Let $\sqrt{2} = a/b$ " with no mention of coprime	The contradiction at the end proves nothing — the closing mark goes
" a^2 is even, so a is even" with no reason	The master key is the middle mark; asserting it does not earn it
Jumping $4c^2 = 2b^2$ straight to " b is even"	The halving line is a step, not arithmetic you may skip
Ending at "both are even" without a conclusion	No $\therefore \sqrt{2}$ is irrational means an unfinished answer

NOW TRY THESE TWO

(i) Prove that $7\sqrt{5}$ is irrational. (ii) Prove that $6 + \sqrt{2}$ is irrational. Both are NCERT Exercise 1.2 Q3.

Hints: for (i) assume $7\sqrt{5} = a/b$, then $\sqrt{5} = a/7b$, which is rational — contradiction. For (ii) assume $6 + \sqrt{2} = a/b$, then $\sqrt{2} = (a - 6b)/b$, which is rational — contradiction. Same engine, both times: isolate the surd.

Turn one clean proof into every mark in the chapter

- **Chapter 1 is free on examfront** — the whole of Real Numbers, including these proofs, at no cost.
- **Mistake identification** tells you whether you lost the mark on the coprime line, the master key or the conclusion.
- **Anytime tests** let you rotate $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ until the recipe is automatic.
- **Weekly tests** keep Real Numbers warm while you move on to Polynomials.
- **Full mock tests before the exam** — the real 38-question, 80-mark, three-hour paper, timed.

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