

Geometrical Meaning of the Zeroes of a Polynomial

$$p(\alpha) = 0 \Leftrightarrow \text{the graph of } y = p(x) \text{ meets the x-axis at } (\alpha, 0)$$

One idea, said two ways

A **zero** is a number α with $p(\alpha) = 0$. Every point on the x-axis has $y = 0$, so a point of the graph sits *on* the x-axis exactly when $p(x) = 0$. Algebra and geometry are describing the same thing.

Algebra says	Geometry says	So in the exam you...
$p(\alpha) = 0$	The curve meets the x-axis at $(\alpha, 0)$	count meeting points, not turns
p has no real zero	The curve never reaches the x-axis	answer "no real zero" – that is complete
$p(0) = c$	The curve crosses the y-axis at $(0, c)$	never count this as a zero

Degree sets the ceiling

Polynomial	Degree	Graph shape	At most
Linear $ax + b$	1	Straight line	1 zero, at $x = -b/a$
Quadratic $ax^2 + bx + c$	2	Parabola	2 zeroes
Cubic $ax^3 + bx^2 + cx + d$	3	Up to two turns	3 zeroes
Degree n	n	–	n zeroes

At most, never *exactly*. $x^2 + 1$ has degree 2 and no real zero at all.

One line, one crossing, one zero

A non-horizontal straight line meets the x-axis exactly once, so **$ax + b$ has exactly one zero**, at $x = -b/a$.

Example. $y = 2x + 3$ has $a = 2$, $b = 3$, so the zero is $-3/2 = -1.5$. Check: $2(-1.5) + 3 = -3 + 3 = 0$ ✓

On the graph the line crosses at **$(-1.5, 0)$** – the x-coordinate of that single crossing point *is* the zero.

A parabola has exactly three possibilities

- Cuts twice** → **two distinct zeroes**. $y = x^2 - 3x - 4$ meets the axis at $(-1, 0)$ and $(4, 0)$. Discriminant 25, positive.
- Touches once** → **one zero (two equal zeroes)**. $y = (x - 2)^2$ just grazes the axis at $(2, 0)$ and turns back. Discriminant 0. This is *one* zero, not two.
- Misses entirely** → **no real zero**. $y = x^2 + 1$ stays above the axis for every x . Discriminant -4 , negative.

What NCERT and CBSE actually ask for

In scope: *reading* a printed graph and stating how many zeroes $p(x)$ has – that is the whole of NCERT Exercise 2.1, and it is the way this topic reaches the paper.

Not in scope: plotting a quadratic or cubic graph yourself; the division algorithm for polynomials, which the rationalised syllabus removed.

Quadratics only: the zeroes-and-coefficients formulas are examinable for quadratic polynomials. The cubic graphs are here to make "at most n " concrete, nothing more.

Key takeaways

- ▶ A zero is an **x-intercept**: the x-coordinate where $y = p(x)$ meets the x-axis.
- ▶ Number of real zeroes = number of points where the curve **touches or crosses** the x-axis.
- ▶ Degree n gives **at most** n zeroes – a degree-2 curve can have 2, 1 or none.
- ▶ A curve that only **touches** the axis has one zero, not two.
- ▶ The y-intercept is $p(0)$. It is **never** a zero unless it happens to be 0.

Worked Example & Exam Practice

Reading zeroes off a graph
NCERT Exercise 2.1 style

Count where the curve **meets** the x-axis — not the bends, not the y-intercept

3 MARKS · WORKED IN FULL

Draw the conclusion for $p(x) = x^2 - 3x - 4$: find its zeroes and say what the graph does.

Step 1 — factorise. $x^2 - 3x - 4 = (x - 4)(x + 1)$, so $p(x) = 0$ when $x = 4$ or $x = -1$.

Step 2 — check both. $p(4) = 16 - 12 - 4 = 0$ ✓ and $p(-1) = 1 + 3 - 4 = 0$ ✓

Step 3 — read it geometrically. The parabola opens upwards ($a = 1 > 0$) and cuts the x-axis at the two points **(-1, 0)** and **(4, 0)**. Two meeting points, two zeroes.

Step 4 — the trap. $p(0) = -4$, so the curve crosses the y-axis at $(0, -4)$. That is the y-intercept, *not* a zero. The lowest point is $(1.5, -6.25)$ — a turn, also not a zero.

Check: sum of zeroes $4 + (-1) = 3 = -b/a$, product $4 \times (-1) = -4 = c/a$. Both match.

NCERT EXERCISE 2.1 · THE ONLY SKILL IT TESTS

You are shown a graph and asked for the number of zeroes. Read it like this:

What the curve does	Zeroes	Why
Never reaches the x-axis	0	$p(x)$ is never 0
Crosses it once	1	one meeting point
Touches it once and turns back	1	one meeting point, two equal zeroes
Cuts it at two separate places	2	two meeting points
Cuts it at three separate places	3	degree is at least 3
Bends twice but cuts once	1	turns are not zeroes

1 MARK · TOUCHES vs CUTS

$y = x^2 - 4x + 4$. Factorising, $x^2 - 4x + 4 = (x - 2)^2$, so the only solution is $x = 2$ and the discriminant is $16 - 16 = 0$. The parabola comes down, *touches* the x-axis at $(2, 0)$ and turns back up.

Answer: one zero. Writing "two zeroes" because the degree is 2, or because the bracket is squared, loses the mark. One meeting point means one zero, described as two equal zeroes.

WHERE THE MARKS ACTUALLY GO

Counting the y-intercept. It is $p(0)$. Only the x-axis carries zeroes.

Counting turning points. A cubic can bend twice and still cut the axis once.

Assuming degree n means exactly n zeroes. It means at most n .

Calling a touch two zeroes. One point on the axis is one zero.

Rejecting a graph that misses the axis. "No real zero" is a complete answer.

NOW TRY THESE

1. Find the zeroes of $p(x) = x^2 - 2x - 8$ and name the two points where its graph meets the x-axis. (Answer: $(x - 4)(x + 2)$, zeroes 4 and -2, points $(4, 0)$ and $(-2, 0)$; $p(0) = -8$ is the y-intercept.)

2. $p(x) = x^3 - x$. How many times does its graph meet the x-axis? (Answer: $x(x - 1)(x + 1)$, so three times — at $(-1, 0)$, $(0, 0)$ and $(1, 0)$. One of the zeroes is the origin itself.)

Turn "I understood it" into marks on examfront

- ▶ **Chapter 1 is free** — start practising today, no payment needed.
- ▶ **Mistake identification** — every wrong answer is tagged, so you see whether it was a concept gap, a sign slip or a misread.
- ▶ **Anytime tests** — sit a chapter test the moment you finish a topic, while it is still fresh.
- ▶ **Weekly tests** — short, regular checks that stop old chapters quietly fading.
- ▶ **Mock tests before the exam** — full 80-mark papers under real timing, so board day is your fourth run, not your first.

examfront.com