Fundamental Theorem of Arithmetic

2-page revision summary — prime factorisation, HCF & LCM. Page 1 of 2: concepts & formulas.

THE THEOREM

Every composite number can be expressed as a product of prime numbers, and this factorisation is **unique** — apart from the order in which the primes occur.

Key definitions

Prime number
A number with exactly two factors: 1 and itself — 2, 3, 5, 7, 11, 13, …

Composite number
A number with more than two factors — 4, 6, 8, 9, 12, … (1 is neither prime nor composite.)

Prime factorisation
Writing a number as a product of primes, in powers-of-primes form with primes in ascending order.

Uniqueness
The same primes appear every time; only the order can change. $2 \times 3 \times 5$ and $5 \times 2 \times 3$ both give 30.

Core formulas

RESULT	STATEMENT
Standard form	$n = p_1^a \times p_2^b \times p_3^c \times \ldots$ where $p_1 < p_2 < p_3$ are primes
HCF	Product of the **smallest** power of each **common** prime
LCM	Product of the **greatest** power of **every** prime that appears
Two-number rule	$\mathrm{HCF}(a, b) \times \mathrm{LCM}(a, b) = a \times b$ — valid for **two** numbers only
Quick examples	$12 = 2^2 \times 3$ · $84 = 2^2 \times 3 \times 7$ · $360 = 2^3 \times 3^2 \times 5$
Ends in 0?	A number ends in 0 only if both **2** and **5** are in its factorisation

Key takeaways

- **One number, one recipe.** However you start the factor tree, you finish with the same primes — the whole chapter rests on that.
- **Split until every leaf is prime.** $60 = 4 \times 15$ is unfinished; $60 = 2^2 \times 3 \times 5$ is the answer.
- **HCF is Common and Conservative** (smallest powers); **LCM is Large and Lavish** (every prime, highest power).
- **Cross-check with HCF × LCM = a × b.** A free 5-second check on every two-number question.
- **Missing prime = whole answer.** 15^n has no 2, so it never ends in 0; 6^n has no 5, so it never ends in 5.
- **Spot the common factor.** $7 \times 11 \times 13 + 13 = 13 \times 78$, so it is composite, not prime.

Worked Example: HCF & LCM by Prime Factorisation

Page 2 of 2 — the exact method examiners expect, step by step.

Question. Find the HCF and the LCM of **336** and **54** by prime factorisation, and verify your answer.

1. **Factorise 336.** $336 = 2 \times 168 = 2 \times 2 \times 84 = 2 \times 2 \times 2 \times 42 = 2 \times 2 \times 2 \times 2 \times 21 = \mathbf{2^4 \times 3 \times 7}$
2. **Factorise 54.** $54 = 2 \times 27 = 2 \times 3 \times 9 = \mathbf{2 \times 3^3}$
3. **Line up the primes and pick the powers.**

PRIME	IN 336	IN 54	HCF TAKES (SMALLEST)	LCM TAKES (GREATEST)
2	2^4	2^1	2^1	2^4
3	3^1	3^3	3^1	3^3
7	7^1	—	not common	7^1

4. **HCF** $= 2 \times 3 = \mathbf{6}$ (only the primes present in *both* numbers, at their smallest power)
5. **LCM** $= 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7 = \mathbf{3024}$ (every prime, at its greatest power — 7 is included even though it is only in 336)

Verify with the two-number rule: HCF × LCM = 6 × 3024 = 18144, and 336 × 54 = 18144. They match ✓
Careful: this check works for two numbers only — never apply it to three.

Try these yourself: prime factorise 96, 150 and 315 in powers-of-primes form, then find HCF(96, 150) and LCM(96, 150) and verify with the product rule.