

Fundamental Theorem of Arithmetic

2-page revision summary — prime factorisation, HCF & LCM. Page 1 of 2: concepts & formulas.

THE THEOREM

Every composite number can be expressed as a product of prime numbers, and this factorisation is **unique** — apart from the order in which the primes occur.

Key definitions

Prime number

A number with exactly two factors: 1 and itself — 2, 3, 5, 7, 11, 13, ...

Composite number

A number with more than two factors — 4, 6, 8, 9, 12, ... (1 is neither prime nor composite.)

Prime factorisation

Writing a number as a product of primes, in powers-of-primes form with primes in ascending order.

Uniqueness

The same primes appear every time; only the order can change. $2 \times 3 \times 5$ and $5 \times 2 \times 3$ both give 30.

Core formulas

RESULT	STATEMENT
Standard form	$n = p_1^a \times p_2^b \times p_3^c \times \dots$ where $p_1 < p_2 < p_3$ are primes
HCF	Product of the smallest power of each common prime
LCM	Product of the greatest power of every prime that appears
Two-number rule	$HCF(a, b) \times LCM(a, b) = a \times b$ — valid for two numbers only
Quick examples	$12 = 2^2 \times 3 \cdot 84 = 2^2 \times 3 \times 7 \cdot 360 = 2^3 \times 3^2 \times 5$
Ends in 0?	A number ends in 0 only if both 2 and 5 are in its factorisation

Key takeaways

- ▶ **One number, one recipe.** However you start the factor tree, you finish with the same primes — the whole chapter rests on that.
- ▶ **Split until every leaf is prime.** $60 = 4 \times 15$ is unfinished; $60 = 2^2 \times 3 \times 5$ is the answer.
- ▶ **HCF is Common and Conservative** (smallest powers); **LCM is Large and Lavish** (every prime, highest power).
- ▶ **Cross-check with $HCF \times LCM = a \times b$.** A free 5-second check on every two-number question.
- ▶ **Missing prime = whole answer.** 15^n has no 2, so it never ends in 0; 6^n has no 5, so it never ends in 5.
- ▶ **Spot the common factor.** $7 \times 11 \times 13 + 13 = 13 \times 78$, so it is composite, not prime.

Worked Example: HCF & LCM by Prime Factorisation

Page 2 of 2 — the exact method examiners expect, step by step.

Question. Find the HCF and the LCM of **336** and **54** by prime factorisation, and verify your answer.

1 Factorise 336. $336 = 2 \times 168 = 2 \times 2 \times 84 = 2 \times 2 \times 2 \times 42 = 2 \times 2 \times 2 \times 2 \times 21 = 2^4 \times 3 \times 7$

2 Factorise 54. $54 = 2 \times 27 = 2 \times 3 \times 9 = 2 \times 3^3$

3 Line up the primes and pick the powers.

PRIME	IN 336	IN 54	HCF TAKES (SMALLEST)	LCM TAKES (GREATEST)
2	2^4	2^1	2^1	2^4
3	3^1	3^3	3^1	3^3
7	7^1	—	not common	7^1

4 HCF $= 2 \times 3 = 6$ (only the primes present in *both* numbers, at their smallest power)

5 LCM $= 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7 = 3024$ (every prime, at its greatest power — 7 is included even though it is only in 336)

Verify with the two-number rule: $\text{HCF} \times \text{LCM} = 6 \times 3024 = 18144$, and $336 \times 54 = 18144$. They match ✓

Careful: this check works for two numbers only — never apply it to three.

Try these yourself: prime factorise 96, 150 and 315 in powers-of-primes form, then find $\text{HCF}(96, 150)$ and $\text{LCM}(96, 150)$ and verify with the product rule.

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