

Forming a Quadratic Equation from a Word Problem

Let → Express → Translate → Simplify • finish at $ax^2 + bx + c = 0$, $a \neq 0$

The four steps, and what each one earns

Step	What you actually write	Why it earns a mark
1. Let	"Let the breadth be x metres."	Names the unknown and fixes the unit. Examiners look for this line.
2. Express	length = $(2x + 2)$ m	Every other quantity now depends on one variable, not two.
3. Translate	$x(2x + 2) = 480$	The given condition – area, product or total – becomes an equation.
4. Simplify	$x^2 + x - 240 = 0$	Standard form is the answer. Stopping at step 3 leaves the last mark.

Why these situations give x^2 at all

A quadratic appears only when the condition **multiplies two expressions that both contain x** . If the condition merely adds multiples of the unknowns, the equation stays linear – that is the Chapter 3 case.

Situation type	Condition used	Where the x^2 comes from
Area	length $\times$ breadth = area	$x \times (2x + 2)$
Product of numbers	first $\times$ second = product	$(x - 4) \times (36 - x)$
Cost / collection	items $\times$ price each = total	$x \times (30 - x)$

Finishing the last line properly

Divide out any common factor. $2x^2 + 2x - 480 = 0$ becomes $x^2 + x - 240 = 0$. Smaller coefficients make the factorisation that follows far easier.

Make the x^2 coefficient positive. $-x^2 + 40x - 396 = 0$ becomes $x^2 - 40x + 396 = 0$ after multiplying through by -1 .

Write descending powers. x^2 term, then x term, then the constant, all on one side with 0 on the other.

NCERT scope: this is Chapter 4, Exercise 4.1 Q2 – “Represent the following situations in the form of quadratic equations”. Its four parts are an area problem, a product of two consecutive integers, an age problem and a train-speed problem. Quadratic Equations sits in **Unit II – Algebra, 20 of the 80 theory marks**.

First, is it even quadratic?

Exercise 4.1 opens by asking you to **check** before you form. Judge only after expanding both sides and collecting terms – never from the way the question is printed.

$$(x + 1)^2 = 2(x - 3) \rightarrow x^2 + 2x + 1 = 2x - 6 \rightarrow x^2 + 7 = 0. \text{ Quadratic.}$$

$$x(x + 1) + 8 = (x + 2)(x - 2) \rightarrow x^2 + x + 8 = x^2 - 4 \rightarrow x + 12 = 0. \text{ Not quadratic – the } x^2 \text{ terms cancel, so } a = 0.$$

Key takeaways

- Let x be the quantity the question asks for – never a quantity you can build from it.
- Express every other quantity in terms of x before you write a single equals sign.
- Area, product and cost conditions all multiply two x -dependent brackets. That is the x^2 .
- The answer to a “represent mathematically” question **is** the standard-form equation.
- Divide out common factors and keep the x^2 coefficient positive on the final line.

Two NCERT Situations, Translated Step by Step

3 MARKS · AREA

The area of a rectangular plot is 528 m^2 . The length is one more than twice the breadth. Form a quadratic equation and find the dimensions.

Let the breadth be x metres. Express: length = $(2x + 1)$ m.

Translate: $x(2x + 1) = 528$ Simplify: $2x^2 + x - 528 = 0$ ← full marks stop here if the question says “represent”.

Solve: $D = 1^2 - 4(2)(-528) = 1 + 4224 = 4225$, and $\sqrt{4225} = 65$, so $x = (-1 \pm 65)/4 = 16$ or -16.5 .

A breadth cannot be negative, so **breadth = 16 m, length = 33 m. Check:** $16 \times 33 = 528 \text{ m}^2$, and $33 = 2(16) + 1$. ✓

THE EXTRA LAYER · AGES

Rohan's mother is 26 years older than him. Three years from now, the product of their ages will be 360. Form a quadratic equation.

Let Rohan's present age be x years, so his mother is $(x + 26)$. Express the shifted ages: in 3 years they are $(x + 3)$ and $(x + 29)$.

Translate: $(x + 3)(x + 29) = 360 \rightarrow x^2 + 32x + 87 = 360 \rightarrow x^2 + 32x - 273 = 0$.

Roots: $D = 1024 + 1092 = 2116$, $\sqrt{2116} = 46$, $x = 7$ or -39 , so Rohan is 7 and his mother 33. **Check:** $10 \times 36 = 360$. ✓ The only new move is **shifting each age by 3 before multiplying**.

PHRASE → ALGEBRA

What the question says	What you write
"two more than twice the breadth"	$2x + 2$ (not $2(x + 2)$)
"two consecutive positive integers"	x and $x + 1$
"they have 40 stamps between them"	x and $40 - x$
"each gives away 4"	$x - 4$ and $36 - x$
"3 years from now"	$x + 3$
"price is 30 minus the number sold"	Rs $(30 - x)$

WHERE THE MARKS ACTUALLY GO

✗ No "Let" line, so the examiner cannot tell what x is. ✗ Reading "two more than twice" as $2(x + 2)$.

✗ Sign slip expanding a bracket such as $(x - 4)(36 - x)$. ✗ Stopping at $x(2x + 1) = 528$ instead of standard form.

✗ Keeping a negative root that the story rules out — a length, an age or a count of items is never negative.

NOW TRY THIS ONE

The sum of the squares of two consecutive positive odd integers is 394. Form the quadratic equation and find them.

Answer: $x^2 + (x + 2)^2 = 394 \rightarrow 2x^2 + 4x - 390 = 0 \rightarrow x^2 + 2x - 195 = 0 \rightarrow (x + 15)(x - 13) = 0 \rightarrow x = 13$, so **13 and 15**. Check: $169 + 225 = 394$. ✓

Practise the translation until it is automatic

- **Chapter 1 is free** — start on examfront without paying anything.
- **Mistake Identification** tells you whether you lose marks on the translation or the expansion.
- **Anytime tests** on any topic, the moment you finish studying it.
- **Weekly tests** keep Chapter 4 alive while you move on to the next chapter.
- **Full mock tests** before the boards, timed and marked like the real paper.

examfront.com