

The Distance Formula

Subtract, square, add, square-root — one rule for the straight-line distance between any two points.

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{and from the origin} \quad OP = \sqrt{x^2 + y^2}$$

Every part of the formula, in plain words

Symbol	What it is	What it does in the formula
$x_1, y_1 \cdot x_2, y_2$	The two points P and Q	Either point can be first — the squares remove the order
$(x_2 - x_1)$	Horizontal gap	One leg of a right triangle
$(y_2 - y_1)$	Vertical gap	The other leg
The square root	Turns two gaps into one length	Keep the positive root — distance is never negative

The rhythm to repeat every single time: **find the gaps** → **square and add** → **square-root**.

It is the Pythagoras theorem in coordinates

Draw a horizontal line from P and a vertical line from Q. They meet at a right angle at a corner T, giving right triangle PTQ with legs $PT = (x_2 - x_1)$ and $QT = (y_2 - y_1)$ and hypotenuse PQ. Then $PQ^2 = PT^2 + QT^2$, and the positive root is the formula.

Why this is worth knowing: you never have to memorise the formula. If you can draw the right triangle, you can rebuild it in ten seconds in the exam hall.

Three shortcuts the formula collapses into

Situation	Use	Example
Distance from the origin	$OP = \sqrt{x^2 + y^2}$	$(8, 15) \rightarrow \sqrt{64 + 225} = 17$
Same y (horizontal line)	$ x_2 - x_1 $	$(2, 5) \text{ and } (9, 5) \rightarrow 7$
Same x (vertical line)	$ y_2 - y_1 $	$(2, 3) \text{ and } (2, 9) \rightarrow 6$

These are not extra formulas — they are what the one formula becomes when a difference is 0.

What the question is really asking for

How the board phrases it	What to set up
"Show the points are collinear"	All three distances; shorter + shorter = longest
"Show it is an isosceles / right triangle"	All three distances; compare, or test $a^2 + b^2 = c^2$
"Find the point on the y-axis equidistant from..."	Take $(0, y)$, set the two distances equal, square both sides
"Find y if the distance is 10"	Square both sides $\Rightarrow$ a quadratic $\Rightarrow$ expect two answers

Key takeaways

- $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — subtract, square, add, square-root.
- It **is** the Pythagoras theorem: the two gaps are the legs, the distance is the hypotenuse.
- From the origin, it shortens to $OP = \sqrt{x^2 + y^2}$.
- Order never matters and distance is never negative — keep only the **positive** root.
- Three points are **collinear** when the two shorter distances add to the longest; otherwise they form a triangle.

5 MARKS · EQUIDISTANT POINT

Find the point on the y-axis which is equidistant from A(6, 5) and B(-4, 3).

Step 1 – use the axis. Every point on the y-axis has $x = 0$, so let the point be $P(0, y)$. *(this line alone carries a mark)*

Step 2 – write both distances. $PA^2 = (6 - 0)^2 + (5 - y)^2$ and $PB^2 = (-4 - 0)^2 + (3 - y)^2$.

Step 3 – set $PA = PB$ and square both sides. $36 + 25 - 10y + y^2 = 16 + 9 - 6y + y^2$.

Step 4 – the y^2 cancels. $61 - 10y = 25 - 6y \Rightarrow 36 = 4y \Rightarrow y = 9$, so the point is **(0, 9)**.

Check: $PA = \sqrt{(36 + 16)} = \sqrt{52} = 2\sqrt{13}$ and $PB = \sqrt{(16 + 36)} = \sqrt{52} = 2\sqrt{13}$. Equal – so (0, 9) is right.

3 MARKS · WHAT KIND OF TRIANGLE?

Show that A(-2, 3), B(8, 3) and C(6, 7) are the vertices of a right-angled triangle.

Step 1 – all three squared distances *(square them; do not root yet – it saves a line and avoids surd errors)*

$$AB^2 = 10^2 + 0^2 = 100 \quad BC^2 = (-2)^2 + 4^2 = 20 \quad CA^2 = (-8)^2 + (-4)^2 = 80$$

Step 2 – test Pythagoras. $BC^2 + CA^2 = 20 + 80 = 100 = AB^2$.

Step 3 – name the right angle. The two shorter sides meet at **C**, so the triangle is right-angled at **C**. Hence proved.

In surd form: $AB = 10$, $BC = 2\sqrt{5}$, $CA = 4\sqrt{5}$ – and these are the **final** answers. CBSE wants exact surds, not decimals.

COLLINEAR, OR A TRIANGLE?

Same test, two outcomes. Find all three distances, then compare.

The three points	The three distances	Verdict
(-1, -1), (2, 3), (5, 7)	5, 5, 10	$5 + 5 = 10 \Rightarrow$ collinear
(1, 1), (2, 3), (3, 5)	$\sqrt{5}$, $\sqrt{5}$, $2\sqrt{5}$	$\sqrt{5} + \sqrt{5} = 2\sqrt{5} \Rightarrow$ collinear
(-2, 3), (8, 3), (6, 7)	$2\sqrt{5}$, $4\sqrt{5}$, 10	$2\sqrt{5} + 4\sqrt{5} \neq 10 \Rightarrow$ triangle

WHERE THE MARKS ACTUALLY GO

- The gaps added instead of squared – the formula has squares because it came from $a^2 + b^2 = c^2$.
- $\sqrt{(9 + 16)}$ turned into $3 + 4$ – a root never splits over a sum; add *inside* first, so it is $\sqrt{25} = 5$.
- A sign lost on a negative coordinate – $(-1) - (-5) = +4$, not -6 .
- The surd rounded off – leave $2\sqrt{13}$ as it is; only simplify ($\sqrt{32} \rightarrow 4\sqrt{2}$), never decimalise.

SPEED KIT · TRIPLES WORTH KNOWING COLD

When the two gaps form one of these, the square root is a whole number and you can write it straight down – no working needed.

Triple	Check	Where it shows up
3, 4, 5 · 6, 8, 10	$9 + 16 = 25$	(1, 2) to (4, 6) $\rightarrow 5$
5, 12, 13 · 15, 36, 39	$25 + 144 = 169$	the 36 km east, 15 km north town $\rightarrow 39$
8, 15, 17 · 7, 24, 25	$64 + 225 = 289$	(8, 15) from the origin $\rightarrow 17$

NOW TRY THIS ONE

The distance between (2, -3) and (10, y) is 10 units. Find y.

Answer: $8^2 + (y + 3)^2 = 100 \Rightarrow (y + 3)^2 = 36 \Rightarrow y + 3 = \pm 6 \Rightarrow y = 3$ or $y = -9$. Squaring gives a quadratic, so **two** answers – dropping one is the classic lost mark here.

Turn the formula into marks – on examfront

- Chapter 1 is free** – start practising today, no payment needed.
- Mistake identification** – see exactly where a squaring slip or a sign error creeps in.
- Anytime tests** – drill distance, collinearity and triangle-type questions in twenty minutes.
- Weekly tests** – keep Coordinate Geometry sharp while you move on to the next chapter.
- Mock tests before the exam** – full papers under real timing, marked by section.

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