

Criteria for Similarity of Triangles

The three shortcut rules – AAA/AA, SSS and SAS – that prove two triangles are similar from just three facts.

$$\triangle ABC \sim \triangle PQR \Rightarrow \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \text{ and } AB/PQ = BC/QR = CA/RP = k$$

What “similar” actually means

Two triangles are **similar** when **both** conditions hold at once: corresponding **angles are equal** and corresponding **sides are in the same ratio**. The three criteria below are shortcuts that prove all six equalities from only three well-chosen facts.

Term	Means	Why it matters
Corresponding angles	Angles at matching vertices	$\triangle ABC \sim \triangle DEF$ fixes $A \leftrightarrow D, B \leftrightarrow E, C \leftrightarrow F$
Corresponding sides	Sides joining matching vertices	AB pairs with DE, not with any side you like
Scale factor k	The one common ratio	Congruence is just the case $k = 1$

Order carries meaning: $\triangle ABC \sim \triangle DEF$ is **not** the same statement as $\triangle ABC \sim \triangle EDF$.

The three criteria at a glance

Rule	What you must show	Reach for it when the question gives you...
AAA / AA	Corresponding angles equal – two are enough	Parallel lines, vertically opposite angles, a shared angle
SSS	$AB/PQ = BC/QR = AC/PR$ – all three	All three sides of both triangles
SAS	$\angle A = \angle P$ and $AB/PQ = AC/PR$	One equal angle plus the two sides that form it

AA is the short form of AAA: fix two angles and the 180° angle sum forces the third.

The one condition students lose marks on

In **SAS** the equal angle must be the **included angle** – the angle sitting **between** the two proportional sides. If you compare AB and AC, the angle you need is $\angle A$. An equal $\angle B$ will not do, however true it is.

Test before you write “SAS”: name the two sides you put in the ratio, then name the vertex they meet at. That vertex is the only angle the rule accepts.

What the question is really asking for

How the board phrases it	What to set up
“DE $\parallel$ BC ... prove the triangles are similar”	Equal angles from the parallels $\Rightarrow$ AA
“Diagonals of a trapezium meet at O ... show $OA/OC = OB/OD$ ”	AA first, then the side ratio
“Sides are 4, 6, 8 and 6, 9, 12 – are they similar?”	Three ratios, simplified $\Rightarrow$ SSS
“ $\angle A = \angle D$ and $AB/DE = AC/DF$ ”	Check A is included $\Rightarrow$ SAS

Key takeaways

- Similar means **equal angles and proportional sides together** – each criterion proves both from three facts.
- AA** is the workhorse: two matching angles are enough, because the third is forced by the 180° sum.
- SSS**: simplify every ratio before you compare. $4/6$ and $6/9$ look different and are both $2/3$.
- SAS** only works with the **included** angle – the one between the two sides you put in the ratio.
- Write the correspondence in matching order; $\triangle ABC \sim \triangle DEF$ means $A \leftrightarrow D, B \leftrightarrow E, C \leftrightarrow F$.

5 MARKS • AA PROOF

ABCD is a trapezium with $AB \parallel DC$. Its diagonals AC and BD meet at O. Prove that $\triangle AOB \sim \triangle COD$, and hence find OC when $AB = 8$ cm, $DC = 12$ cm and $OA = 6$ cm.

Given: $AB \parallel DC$; diagonals meet at O. To prove: $\triangle AOB \sim \triangle COD$.

Proof:

$\angle OAB = \angle OCD$ (alternate angles, $AB \parallel DC$ with transversal AC)

$\angle OBA = \angle ODC$ (alternate angles, $AB \parallel DC$ with transversal BD)

$\angle AOB = \angle COD$ (vertically opposite angles)

$\therefore \triangle AOB \sim \triangle COD$ (AAA / AA criterion) — Hence proved.

Then: $OA/OC = AB/DC$ (corresponding sides of similar triangles) $\Rightarrow 6/OC = 8/12 \Rightarrow OC = 6 \times 12 / 8 = 9$ cm. Check: $6/9 = 2/3$ and $8/12 = 2/3$ — the same k, so the answer is exact.

3 MARKS • SAS NUMERICAL

In $\triangle ABC$ and $\triangle PQR$, $\angle A = \angle P = 60^\circ$, $AB = 5$ cm, $AC = 7.5$ cm, $PQ = 8$ cm and $PR = 12$ cm. Are the triangles similar? If $QR = 9.6$ cm, find BC.

Step 1 — is the equal angle included? $\angle A$ lies between AB and AC; $\angle P$ lies between PQ and PR. Yes.

Step 2 — the two ratios: $AB/PQ = 5/8$ and $AC/PR = 7.5/12 = 5/8$.

Step 3: equal included angles + equal ratios $\Rightarrow \triangle ABC \sim \triangle PQR$ (SAS), with $k = 5/8$.

Step 4: $BC/QR = 5/8 \Rightarrow BC = 9.6 \times 5 / 8 = 6$ cm. Check: $6/9.6 = 0.625 = 5/8$, matching the other two ratios.

SSS • SPOT THE ODD PAIR

Simplify every ratio **before** you decide. One ratio out of line is enough to kill similarity.

The two triangles	The three ratios	Verdict
4, 6, 8 and 6, 9, 12	$2/3, 2/3, 2/3$	Similar by SSS, $k = 2/3$
3, 4, 5 and 6, 8, 10	$1/2, 1/2, 1/2$	Similar by SSS, $k = 1/2$
4, 6, 8 and 6, 9, 11	$2/3, 2/3, 8/11$	Not similar — one ratio differs

WHERE THE MARKS ACTUALLY GO

- Vertices written out of order — the correspondence line itself carries a mark.
- "SAS" quoted with a non-included angle — the whole proof is then unearned.
- Ratios left unsimplified, so two equal ratios are declared different.
- The criterion never named — state *which* rule you used, on its own line.

30-SECOND DRILL • WHICH RULE?

- "PQ $\parallel$ RS, and PS and QR cross at O" $\rightarrow$ equal alternate angles + vertically opposite $\Rightarrow$ AA.
- " $\angle B = \angle E$, $AB/DE = BC/EF$ " $\rightarrow$ B sits between AB and BC $\Rightarrow$ SAS (here $\angle B$ is included).
- " $\angle B = \angle E$, $AB/DE = AC/DF$ " $\rightarrow$ A, not B, is between AB and AC $\Rightarrow$ SAS does not apply.

NOW TRY THIS ONE

In $\triangle PQR$, $\angle P = 50^\circ$ and $\angle Q = 60^\circ$. In $\triangle XYZ$, $\angle Y = 60^\circ$ and $\angle Z = 70^\circ$. Are they similar — and if so, write the correspondence correctly.

Answer: $\angle R = 70^\circ$ and $\angle X = 50^\circ$, so $P \leftrightarrow X, Q \leftrightarrow Y, R \leftrightarrow Z \Rightarrow \triangle PQR \sim \triangle XYZ$ by AA.

Turn the rules into marks — on examfront

- **Chapter 1 is free** — start practising today, no payment needed.
- **Mistake identification** — see exactly where correspondence or the included angle slipped.
- **Anytime tests** — drill AA, SSS and SAS whenever you have twenty minutes.
- **Weekly tests** — keep Triangles sharp while you move on to the next chapter.
- **Mock tests before the exam** — full papers under real timing, marked by section.

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