

Consistent and Inconsistent Pair of Linear Equations

Chapter 3 · Pair of Linear Equations in Two Variables · page 1 of 2 — the concept

Write both as $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then compare a_1/a_2 , b_1/b_2 , c_1/c_2

Three words you must be able to define

Term	Means	The lines	Solutions
Consistent	Has at least one solution	Intersect or coincide	One, or infinitely many
Inconsistent	Has no solution at all	Parallel, never meet	None
Dependent	The two equations are equivalent – one is a multiple of the other	Coincident: the same line drawn twice	Infinitely many (so always consistent)

Memory hook: consistent pairs agree somewhere, inconsistent pairs never agree, dependent pairs are clones.

The ratio conditions — the whole chapter in three rows

Compare the ratios	Lines look like	Number of solutions	Name of the pair
$a_1/a_2 \neq b_1/b_2$	Intersecting	Exactly one (unique)	Consistent
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident	Infinitely many	Dependent (consistent)
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel	None	Inconsistent

Why it works: a_1/a_2 and b_1/b_2 compare the *directions* of the two lines. Different directions → they must cross once. Same direction → c_1/c_2 decides whether it is the same line or a parallel copy.

How to apply the test — four steps

1. Rewrite **both** equations as $ax + by + c = 0$, moving every term to one side.
2. Read off a_1 , b_1 , c_1 and a_2 , b_2 , c_2 **with their signs**.
3. Compute a_1/a_2 and b_1/b_2 . Unequal → unique solution, consistent. **Stop here**.
4. Equal? Then compute c_1/c_2 . All three equal → infinitely many (dependent); c-ratio different → no solution (inconsistent).

Sign trap: given $2x + 5y = 10$, the constant is $c = -10$, not $+10$. This one slip is the most common wrong answer in the whole topic.

What the question is really asking for

If the question says...	Set up this condition
unique solution · intersecting lines	$a_1/a_2 \neq b_1/b_2$
no solution · parallel lines · inconsistent	$a_1/a_2 = b_1/b_2 \neq c_1/c_2$
infinitely many · coincident lines · dependent	$a_1/a_2 = b_1/b_2 = c_1/c_2$

Key Takeaways

- **Consistent** = at least one solution; **inconsistent** = none; **dependent** = same line, infinitely many, always consistent.
- The three-ratio test decides everything — no graph is needed.
- Always convert to general form ($= 0$) first; sign errors in c cause most wrong answers.
- For a unique solution **c_1/c_2 is irrelevant**; it only matters once the first two ratios match.
- The same test powers every **find-k** board question.

Worked Examples — and the k-value twist

Page 2 of 2 — the ratio test run on real board-style questions

WORKED EXAMPLE · 3 MARKS

Find the value of k for which $2x + 3y - 5 = 0$ and $kx - 6y + 8 = 0$ has (i) a unique solution, (ii) no solution. (iii) Can it ever have infinitely many?

Step 1. Both are already in general form: $a_1 = 2$, $b_1 = 3$, $c_1 = -5$ and $a_2 = k$, $b_2 = -6$, $c_2 = 8$.

Step 2. $b_1/b_2 = 3/(-6) = -1/2$ and $c_1/c_2 = -5/8$.

(i) **Unique solution** needs $a_1/a_2 \neq b_1/b_2$, i.e. $2/k \neq -1/2$, i.e. $4 \neq -k$, so $k \neq -4$ (and $k \neq 0$, or the ratio is undefined).

(ii) **No solution** needs $a_1/a_2 = b_1/b_2 \neq c_1/c_2$. From $2/k = -1/2$ we get $k = -4$. Check the c-ratio: $-1/2 = -0.5$ but $-5/8 = -0.625$, so they differ — the condition holds.

(iii) Infinitely many would need $b_1/b_2 = c_1/c_2$, i.e. $-1/2 = -5/8$, which is false whatever k is. **No value of k gives infinitely many solutions.**

Check: at $k = -4$ the second equation is $-4x - 6y + 8 = 0$, while $-2 \times$ the first gives $-4x - 6y + 10 = 0$ — same left side, different constant. Parallel. Confirmed.

SPEED DRILL · CLASSIFY IN 20 SECONDS

Pair	Ratios	Verdict
$3x + 2y = 5$ $2x - 3y = 7$	$3/2$ vs $2/(-3)$	$a_1/a_2 \neq b_1/b_2 \rightarrow$ consistent, unique solution
$x - 2y = 8$ $3x - 6y = 24$	$1/3 = 1/3 = 1/3$	All three equal $\rightarrow$ dependent, infinitely many
$5x - 4y + 8 = 0$ $10x - 8y + 9 = 0$	$1/2 = 1/2$ but $8/9$	c-ratio differs $\rightarrow$ inconsistent, no solution

WHERE THE MARKS ACTUALLY GO

1. Not converting to $= 0$ form first, so c is written with the wrong sign — every ratio after it is wrong.
2. Testing c_1/c_2 when $a_1/a_2 \neq b_1/b_2$ has already settled it — wasted time, and sometimes a wrong answer.
3. Losing a minus sign: $-1/-2$ is $+1/2$, and that flips coincident into intersecting.
4. Stating k without checking the c-ratio in a "no solution" question — the check line is worth a mark.

NOW TRY THIS ONE

For what value of k does the pair $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ have **no solution**?

Answer: $3/(2k - 1) = 1/(k - 1)$ gives $3k - 3 = 2k - 1$, so $k = 2$. *Check:* at $k = 2$ the b-ratio is 1 and the c-ratio is $1/5$, which differ — so the lines really are parallel.

THE ONE EDGE CASE · A ZERO COEFFICIENT

The ratios need a_2 , b_2 , c_2 non-zero; a zero on top is fine. **Example.** $x = 4$ and $2x + 3y = 5$ give $a_1/a_2 = 1/2$ and $b_1/b_2 = 0/3 = 0$ — unequal, so a **unique solution**, and indeed $8 + 3y = 5$ puts it at $(4, -1)$.

Safe in every case: the pair has exactly one solution when $a_1b_2 - a_2b_1 \neq 0$. That form never divides by zero, so use it if a denominator vanishes.

Make the ratio test automatic — on examfront

- **Chapter 1 is free** — start practising today, no payment needed.
- **Mistake Identification** pinpoints the exact step you slip on: sign of c , or the c-ratio check.
- **Anytime tests** on any topic, the moment you finish studying it.
- **Weekly tests** that keep old chapters alive, not fading.
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