Basic Proportionality Theorem (Thales Theorem)

A line parallel to one side of a triangle cuts the other two sides in the same ratio — the result the whole similarity chapter is built on.

The theorem (Theorem 6.1)

If a line is drawn **parallel to one side** of a triangle to intersect the other two sides in **distinct points**, then it divides those two sides in the **same ratio**.

In △ABC, if DE ∥ BC (D on AB, E on AC) ⟹ AD/DB = AE/EC

Corollary forms (all true when DE ∥ BC): AD/AB = AE/AC · DB/AB = EC/AC · AB/AD = AC/AE

The converse (Theorem 6.2) — used to prove parallelism

If a line divides **any two sides of a triangle in the same ratio**, then that line is **parallel to the third side**.

AD/DB = AE/EC ⟹ DE ∥ BC

You are given…	…so you use
A parallel line, asked for a length	**BPT** — write AD/DB = AE/EC, substitute, solve
Four lengths, asked "is it parallel?"	**Converse** — compute both ratios, compare

Proof (area method) — the exam-ready version

Given: △ABC with DE ∥ BC, D on AB, E on AC. **To prove:** AD/DB = AE/EC.

Construction: join BE and CD; draw DM ⊥ AC and EN ⊥ AB.

1. Same height EN: ar(ADE) = ½ × AD × EN, ar(BDE) = ½ × DB × EN ⟹ **ar(ADE)/ar(BDE) = AD/DB**

2. Same height DM: ar(ADE) = ½ × AE × DM, ar(DEC) = ½ × EC × DM ⟹ **ar(ADE)/ar(DEC) = AE/EC**

3. △BDE and △DEC share base **DE** and lie between the same parallels **DE and BC** ⟹ **ar(BDE) = ar(DEC)**

4. Equal numerators over equal denominators ⟹ **AD/DB = AE/EC** ∎

Key Takeaways

- **BPT = Thales Theorem.** Parallel line ⟶ equal ratios; the converse runs it backwards.
- The mark-earning line of the proof is **step 3** — "same base DE, between the same parallels" — never skip it.
- **Match your pieces:** AD/DB = AE/EC (part-to-part) or AD/AB = AE/AC (part-to-whole). Never mix the two.
- **Simplify before comparing** ratios: 4/4.5 and 8/9 look different but are equal.
- BPT is the tool used to prove the **similarity criteria** (AAA, SSS, SAS) and the **Mid-point Theorem**.

Worked examples

Example 1 — find a missing length (uses BPT)

Question. In △ABC, DE ∥ BC with D on AB and E on AC. If AD = 1.5 cm, DB = 3 cm and AE = 1 cm, find EC.

Step 1 — quote the theorem. DE ∥ BC is given, so by BPT: AD/DB = AE/EC.

Step 2 — substitute. 1.5/3 = 1/EC

Step 3 — cross-multiply. 1.5 × EC = 3 × 1 = 3

EC = 3 ÷ 1.5 = 2 cm

Check: AD/DB = 1.5/3 = 0.5 and AE/EC = 1/2 = 0.5 ✓ — the ratios match, as BPT requires.

Example 2 — is the line parallel? (uses the converse)

Question. In △PQR, E lies on PQ and F on PR, with PE = 4 cm, EQ = 4.5 cm, PF = 8 cm, FR = 9 cm. Is EF ∥ QR?

Side PQ	Side PR
PE/EQ = 4/4.5 = **8/9**	PF/FR = **8/9**

Ratios equal ⟹ by the converse of BPT, EF ∥ QR

Contrast case. If PE = 3.9, EQ = 3, PF = 3.6, FR = 2.4 then PE/EQ = 1.3 but PF/FR = 1.5. Unequal ⟹ **EF is not parallel to QR**. Always compute both ratios — never judge from the diagram.

Now try this one

In △ABC, DE ∥ BC with AD = 4 cm, AB = 10 cm and AE = 6 cm. Find AC.

Hint: you are given a **whole side** (AB), so use the part-to-whole corollary AD/AB = AE/AC, not AD/DB = AE/EC.

Answer: 4/10 = 6/AC ⟹ AC = 15 cm (so EC = 9 cm).