

Arithmetic Progressions — Concept Summary

Two formulas carry this entire chapter. Learn when each one applies and the marks follow.

What an AP is

An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the term before it. That fixed number is the **common difference, d**.

$$a, a + d, a + 2d, a + 3d, \dots$$

Here **a** = first term, **d** = common difference, **n** = number of terms.

Test for an AP: $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots = d$. If every consecutive difference is the same, it is an AP.

The two core formulas

NTH TERM (GENERAL TERM)

$$a_n = a + (n - 1)d$$

SUM OF FIRST N TERMS

$$S_n = n/2 [2a + (n - 1)d]$$

Useful variant: when the last term **l** is known, $S_n = n/2 (a + l)$. Also $a_n = S_n - S_{n-1}$, and $d = a_n - a_{n-1}$.

Which formula does the question want?

THE QUESTION SAYS...	USE
Find the 20th term / which term is 143 / find a or d	$a_n = a + (n - 1)d$
Find the sum of the first n terms / total savings / total seats	$S_n = n/2 [2a + (n - 1)d]$
First and last term given, sum wanted	$S_n = n/2 (a + l)$
Is this sequence an AP?	Check consecutive differences

Key takeaways

- ▶ **d can be negative.** In 24, 21, 18, ... $d = -3$. Carry the minus sign right through the substitution — sign slips cost the most marks here.
- ▶ **Read "term" vs "sum" carefully.** "The 15th term" and "the sum to 15 terms" need different formulas.
- ▶ **Use $(n - 1)d$, not nd .** Writing $a + nd$ for the nth term is the most common formula error here.
- ▶ **n must be a positive whole number.** If n comes out as 7.5 or negative, that term is not in the AP — say so.
- ▶ **In word problems, write "a = ..., d = ..." first.** It earns method marks even if the arithmetic later slips.

Worked Example — Both Formulas, One Question

Question

In the AP 5, 11, 17, 23, ...

(a) Which term of this AP is 143? (b) Find the sum of its first 20 terms.

Solution

1 Write down a and d.

$a = 5$. $d = 11 - 5 = 6$. Check: $17 - 11 = 6$ and $23 - 17 = 6$, so it is an AP with $d = 6$.

2 (a) Use the nth-term formula — the question asks "which term".

$$a_n = a + (n - 1)d \Rightarrow 143 = 5 + (n - 1)(6)$$

3 Solve for n.

$$143 - 5 = 6(n - 1) \Rightarrow 138 = 6(n - 1) \Rightarrow n - 1 = 23 \Rightarrow n = 24$$

n is a positive whole number, so 143 really is a term of this AP.

(a) 143 is the 24th term.

4 (b) Now the question asks for a sum — switch formulas.

$$S_n = n/2 [2a + (n - 1)d] \text{ with } n = 20, a = 5, d = 6$$

5 Substitute and simplify.

$$S_{20} = 20/2 [2(5) + (20 - 1)(6)] = 10 [10 + 19 \times 6] = 10 [10 + 114] = 10 \times 124$$

(b) $S_{20} = 1240$

Cross-check (b): $a_{20} = 5 + 19(6) = 119$, so $S_{20} = n/2 (a + l) = 10(5 + 119) = 1240$ ✓ — the other variant agrees. A 10-second check worth doing in the exam.

Mistake watch: writing $143 = 5 + 6n$ (forgetting the -1) gives $n = 23$ — one term off, and full marks lost on a question you understood. Always substitute $(n - 1)d$.

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